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Birdsong is the product of the controlled generation of sound embodied in a neuromotor system. From a biophysical perspective, a natural question is that of the difficulty of producing birdsong. To address this, we built a biomimetic syrinx consisting of a stretched simple rubber tube through which air is blown, subject to localized mechanical squeezing with a linear actuator. A large static tension on the tube and small dynamic variations in the localized squeezing allow us to control transitions between three states: a quiescent state, a periodic state and a solitary wave state. The static load brings the system close to threshold for spontaneous oscillations, while small dynamic loads allow for rapid transitions between the states. We use this to mimic a variety of birdsongs via the slow–fast modulated nonlinear dynamics of the physical substrate, the syrinx, regulated by a simple controller. Finally, a minimal mathematical model of the system inspired by our observations allows us to address the problem of song mimicry in an excitable oscillator for tonal songs.

## 1. Introduction

The planning, execution, learning and evolution of birdsong has inspired scientists from a range of fields with an interest in understanding its origins, ethology, neurobiology and biophysics [1–6]. Anatomically, the syrinx, the vocal organ of the bird, contains the vibratory tissues that generate sound and lies at the junction of the bronchi and trachea, and is ensconced in the interclavicular airsac [4,7]. Morphologically, there is a diversity of syrinxes across species but calls and songs arise from a simple universal process: air flow generated by the air-sac couples to the vibrating structures (lateral vibratory masses (LVMs) and tympaniform membranes (MTMs) in non-songbirds and medial vibratory mass (MVM) and medial labium (ML) in songbirds) [4] under active control by the muscles that surround the syrinx [8,9]. Dynamically, the vibrating tissues are viscoelastic materials, composed largely of extracellular matrix materials such as collagen, and thought to determine the fundamental frequency of the song by changes in their tension due to active muscular contractions [10] and passive airsac pressure [11].

To understand how birds use their syrinx requires that we take it apart functionally and morphologically, understand the function of the parts individually, and then synthesize this into a complete explanation. Experiments with excised syrinxes from zebra finches show that they are nonlinear dynamical devices—air flow fed from a pressure source attached to its posterior end causes it to become acoustically active with a complex harmonic spectrum that eventually becomes chaotic above a critical driving pressure [12]. In addition to the nonlinearities embodied in the geometry and elasto-hydrodynamics of slender vibrating structures, biomechanical studies of the stress–strain relation of the vibrating structures show that they are also materially nonlinear [10,13]. However, the role of nonlinear material properties in sound production is debated, and it is thought efficacious to avoid these nonlinearities to achieve precise control of complex songs [14].

A complementary approach asks if it is possible to recreate song artificially by building a mechanical device with enough freedom to be able to reproduce birdsong. Mechanical models that take this route [15] and are capable of phonation above critical pressures. These anatomically inspired mechanical mimics of the syrinx use a combination of soft and stiff elastic tubes [15] and have the ability to phonate and produce complex sounds resembling calls. Recent work [16] shows that controlling bronchial and airsac pressure along with external muscle stimulation on an excised syrinx can be used to control the fundamental frequency, which can then be modulated. However, to produce birdsong in real time requires a fast actuation mechanism, something that has not yet been achieved. These experimental approaches serve as a minimal means to understanding the basic physical and engineering principles behind the production and control of complex birdsong, by focusing only the bare essentials of biological morphology and physiology.

Simultaneously, observations and experiments have been the inspiration for theoretical models to understand sound production. They are mathematically couched in terms of low-order dynamical systems that model the syrinx as an uncontrolled nonlinear self-excited oscillator [17,18]. Over time, by taking note of the myoelastic-aerodynamic (MEAD) mechanism used to explain human speech [19] and birdsong [16], these models have evolved [20] to incorporate the effects of muscle tension and lung pressure and can recreate certain aspects of complex songs [21]. However these lumped-parameter perspectives are unable to account for the fact that a 50-fold increase in membrane tension is required to explain the range of frequency shifts observed in a zebra finch, while mechanical measurements performed on an excised syrinx show that an applied strain as large as 30% changes the effective spring constant only by a factor of 2 [13] (and thus the frequency be just 40%). Furthermore, although enhanced mathematical models with parameters resembling measured ventral muscle activity and airsac pressure [22] from *in vivo* experiments have been used to create zebra finch songs [21], the physical parameters used correspond to a linear spring constant of  $10^{-5} \text{ N m}^{-1}$  and a mass of  $4 \times 10^{-9} \text{ g}$  [23] associated with a time scale  $\approx 10 \mu\text{s}$  that is not consistent with the ability to control such songs [6]. All together, these studies suggest that much still needs to be done to understand the biophysical aspects of controllable birdsong.

Here we describe a biologically inspired physical model of the syrinx with real time control and demonstrate its ability to create and control complex birdsong. Our biomimetic syrinx is an elastic tube under biaxial tension with air flowing through it, and a mechanical probe that can change the tension in the tube dynamically. We show that when this artificial syrinx is loaded with a large static load, and then subject to a superposition of small dynamic loads, we can trigger and quench aeroelastic instabilities that create sound without evoking material nonlinearities, and mimic the complex vocal repertoire of a variety of songbirds. A minimal mathematical model of the system inspired by our observations allows us to address the problem of song mimicry in a linearized excitable oscillator for tonal songs. Our results suggest that the complexity of birdsong may be delegated to the dynamics of the physical substrate, the syrinx, using a large stationary load and a small dynamical one that is regulated by a simple controller.

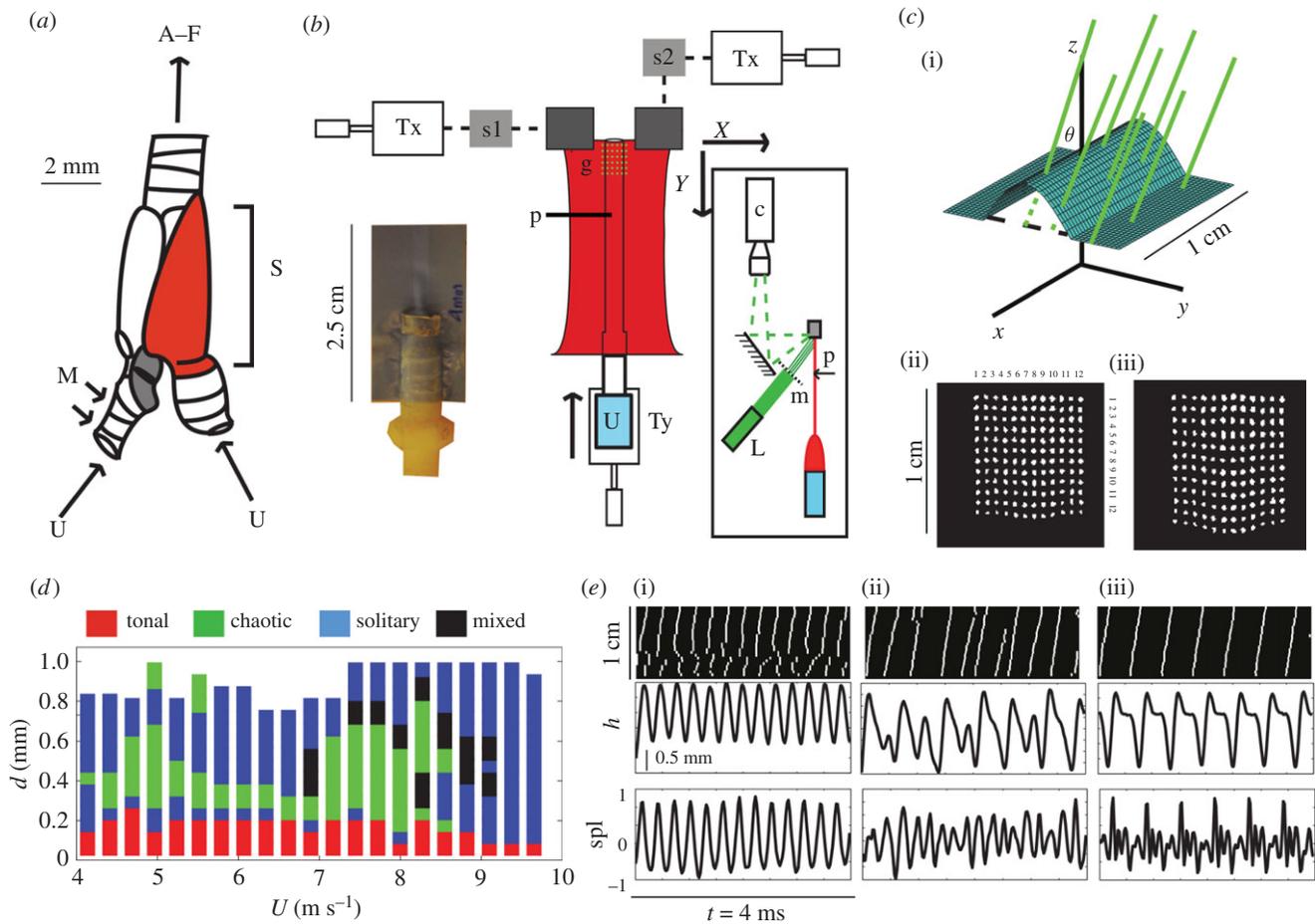
## 2. Controllable biomimetic syrinx

The singing apparatus of songbirds like the zebra finch consists of the airsac, which acts as a pressure-controlled air supply [24], the bronchus, a thin tube that connects the airsacs to the syrinx, which contains the vibrating sound source (the ML and MVM) [4,13], and finally the acoustic filter formed by the tongue and beak [25]. In songbird species, thick muscles surround the syrinx and have a direct effect on the sound features as shown in muscle stimulation studies [16,26] but the exact mechanism of how muscle action and sound activity are related is still only partly understood. These observations serve as the backdrop for a minimal controllable artificial syrinx, which while inspired by earlier physical and mathematical models [15,27], differ in a fundamental way by incorporating a new method of scale separation of loading and control which allows for real-time birdsong production.

### 2.1. Physical model

To mimic the vibrating structures of the syrinx and the bronchus (figure 1*a*), we used a soft thin elastic tube (Young's modulus  $\approx 1 \text{ MPa}$ ) of length 2 cm, outer diameter 2.5 mm and thickness 0.1 mm shown in figure 1*b* (also electronic supplementary materials, figures S1,S2). Our physical model deliberately seeks to avoid the complexity of real bronchi and the layered nature of the MTM, ML and MVM, but allows us to focus on understanding the controllable elastohydrodynamics of phonation. Furthermore, our device is scaled up relative to the zebra finch syrinx, which has a size of  $O(1 \text{ mm})$ , and so its physical properties such as the bending modulus need to be scaled up accordingly to replicate the range of frequencies observed in nature. The dynamic action of the airsac was mimicked by a velocity-controlled air meter (flow rate  $\approx 10^{-5} \text{ m}^3 \text{ s}^{-1}$ ) attached to the caudal end of the tube, while the cranial end was open to the atmosphere and free to vibrate. In reality, the airsac controls the pressure so that controlling flow velocity in the experiment is a simplification, but valid in the limit that the impedance of the bronchus is larger than that of the syrinx. Operating in the velocity-controlled mode ensures that the device does not behave like a pressure-controlled valve.

Song production requires the use of muscles around the syrinx to control the frequency content. For example, in the zebra finch, it has been observed that the major muscles syringealis ventralis, tracheobronchialis ventralis and tracheobronchialis dorsalis [28] directly control the position and tension of the MVM and LL (lateral labia) and reorient the syrinx to a basal tensile state before phonation [8]. In our set-up, translation stages attached to the rear and front of the device (figure 1*b*) allowed us to adjust global longitudinal and lateral tensions to set the operation point of the device in a manner similar to actual songbirds. The translation stages are large and cannot be actuated quickly so can only provide the basal tensile state and slow control of global tension. The rapid changes in song frequency and amplitude, characteristic of finch songs, were induced by the action of a linear motor that presses down on the rubber tube and squeezes it at a point along its length (figure 1*b*; electronic supplementary material, figures S1,S2). The linear motor can be actuated rapidly at frequencies up to 30 Hz with amplitudes of up to 1 mm allowing us to



**Figure 1.** A bird syrinx, its biomimetic analogue, and the dynamics of the device. (a) Schematic of a zebra finch syrinx. S is the sound box (syrinx) of the bird, U is airflow coming from the lungs, A–F is the acoustic filter formed by the oropharyngeal–oesophageal cavity, beak and trachea [29], M denotes muscular action along the length of the bronchus. (b) Schematic of experimental set-up. s1, Tx, strain gauge and stage to control lateral tension; s2, Ty, gauge and stage to control longitudinal tension; U, flowmeter; g, grid projected onto device. Inset (on right) shows side view of the set-up. L, 10 mW Green laser pointer; m, mask used to produce grid; p, probe used to actuate device; c, high-speed video camera (up to 90 kHz frame rate). The probe p is a linear motor to excite the membrane by pushing anywhere along its length, and mimics muscular actuation along the length of the bronchus. An image of the actual device is shown on the left. (c)(i)–(iii) Demonstration of the principle of the optical reconstruction. (i) shows the top surface of the device, described by the function  $z(x, y)$ , being impinged by pencils of laser light described by the functions  $z_n(x, y)$ , where the index  $n$  characterizes the raster location. The light is incident on the device at an angle  $\theta$  and the dashed lines show the intended path of the laser before it is intercepted by the surface. (ii), (iii) Images of the raw data with the flow turned off and on, respectively. The distance that each point in (c)(iii) has moved from its original location in (c)(ii) is proportional to the height at the observed point and this information can be used to reconstruct the entire surface in three dimensions. (d) The phase space of a device as a function of flow velocity  $U$  and probe depth  $d$  measured relative to the first depth (100  $\mu\text{m}$ ) at which sound is produced. Note that the solitary regime may have further subharmonic bifurcations A ‘mixed’ phase between chaotic and solitary also exists, marked in black in the phase space. (e) Digitized kymograph (positive  $y$ -direction corresponds to, height  $h(t)$  of the membrane at a point  $y = 0.5$  cm from the mouth, and sound pressure level (spl) for the three regimes: tonal, aperiodic and solitary. (Online version in colour.)

reproduce rapidly varying transitions in complex songs. In our flow-driven model, there is no acoustic filter; to the first approximation, this has negligible feedback on the vibrating surface and can be omitted. To capture the physical mechanism of sound production and the sound itself, we visualized the surface profile of the oscillating tube at 100 kHz using a three-dimensional direct imaging technique (figure 1c; electronic supplementary material), while simultaneously recording the sound with a microphone.

## 2.2. Uncontrolled passive modes

The onset of phonation requires a critical flow velocity in the range  $0\text{--}8\text{ m s}^{-1}$ , that is a function of the global applied tension in the tube, which ranges from 0 to 100 mN. Even without any localized application of stress via the linear probe, the biomimetic syrinx produced sound when the

global tension crosses a threshold value, even at low velocities ( $0.5\text{ m s}^{-1}$ ). In this regime associated with global modes of the whole tube, the frequency of sound scaled weakly with the applied tension (between 2 and 3 kHz). Bird-song typically includes large and rapid swings in frequency, so that the set point of the device was adjusted to avoid exciting these global self-oscillation modes over the range of flow velocities used. The application of localized constriction via the linear actuator causes changes in the tension while also causing the local air velocities to increase. As a result, the transmural pressure drops. This couples with elastic modes of the tube to create self-sustained oscillations that produced sound with a large range in frequency (fundamental frequency 1–5 kHz, harmonics up to 20 kHz were observed); similar modes of oscillation have also been seen when air flows through un-tensioned soft cylindrical shells [30]. In the presence of air flow, the localized squeezing of the tube

yields a rich repertoire of sounds, including tonal, bursting and chaotic behaviour as a function of probe depth and flow velocity.

For small values of the displacement by the linear motor, the device produces sinusoidal modes as shown in figure 1*d,e* (see also electronic supplementary materials, figure S2*a*). These appear as travelling waves emanating from the probe location and propagate downstream (electronic supplementary material, Video S1). The static position of the probe controls the frequency of the sound produced; sweeping through frequency can be achieved via small dynamical displacements superposed on this static position (figure 2*e*; electronic supplementary material) as the frequency is very sensitive to probe position. These observations suggest that simple auditory gestures can be created by moving the linear actuator along a prescribed path as a function of time, at a rate (30 Hz) much slower than the song frequency (1 kHz) itself (electronic supplementary materials, Audio S1). When the probe presses further down into the device a new mode of oscillation appears, wherein a short section of the tube of length comparable with its radius forms a localized buckle that rapidly travels towards the frontal end of the tube at steady velocity of  $35 \pm 5 \text{ m s}^{-1}$  (electronic supplementary materials, figure S4, Video S3). The velocity of these localized pulses shows only a weak dependence on probe displacement or flow velocity. Following a very short pause (approx. 1 ms, see figure 1*e*(iii); electronic supplementary materials, figures S3,S4) during which the tube regains its original cylindrical shape, the cycle repeats itself. This is suggestive of the system being close to an excitability threshold, with a refractory period characteristic of an excitable medium [31]. In this regime of oscillation, the fundamental frequency and the harmonic structure of the sound are determined by the duration of the pulses which can be controlled by the position of the linear probe (electronic supplementary materials, figure S4). In this same regime, further complex sounds sometimes arise via period doubling bifurcations (see electronic supplementary materials, figure S3*b*, Video S2), with some qualitative similarities to the structure of certain zebra and bengalese finch song motifs [22]. As the probe indentation depth is further increased (figure 1*d,e*), we see a third regime wherein chaotic sounds are often heard, as in many zebra finch songs [32]. This phase is characterized by solitary pulses with stochastic refractory times between pulses as shown in figure 1*e*(ii). Indenting the probe into the tube still further eventually chokes the flow and causes the tube to burst.

Collectively, our observations bear a strong resemblance to self-excited hydroelastic modes observed in a class of devices called Starling resistors [33,34] that have been used to study bioinspired fluid–structure instabilities in the context of Korotkoff sounds [35]. However, there are also some critical differences that are linked to the ability to excite and extinguish sound production using a linear actuator that controls the appearance of dynamical signatures associated with the periodic, excitable and solitary modes of oscillation. Moving through these regimes of simple tonal oscillations and self-sustained localized oscillations is possible by varying the position of the probe by less than 10% and thus allows for a range of sounds via transitions between these phases in a manner that is qualitatively similar to fine muscle control over song. The tonal phase can be used to reproduce sounds from birds like a canary or vireo, while the bursting

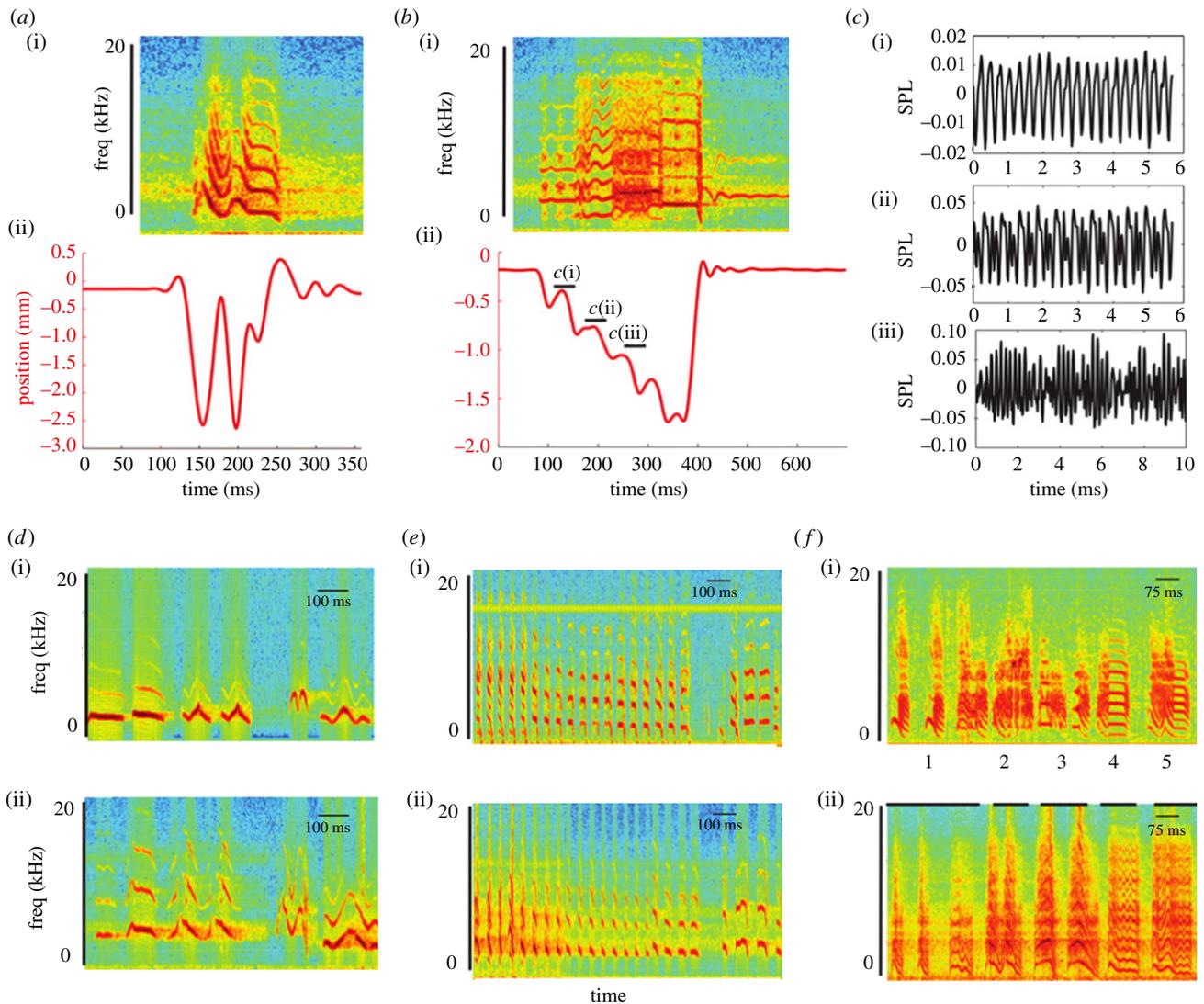
phase is required to mimic sounds produced by birds like the zebra or bengalese finch. Our observations suggest a mechanism for the real-time programming of a song sequence into the dynamics of the linear motor using the modular design and control of the biomimetic syrinx, whereby the global stretch of the tube brings the system close to threshold for spontaneous oscillations, while the rapidly controllable small strains ( $\approx 0.01$ ) controls the onset of oscillation. This allows for the flexibility required in switching modes of phonation and thence song production.

We pause to emphasize that there are many detailed differences between an actual syrinx and our simple physical abstraction. For example, in a song bird the vibrating structure (MVM) is rather short (1 mm) and soft (1 kPa) compared with our long (1 cm) tube-like device made of a much stiffer elastomer. This leads to an absolute difference in the wave speeds observed in the two systems, but can be reconciled by the fact that the bending wave speed scales as  $c \sim (1/r)\sqrt{Et^2/\rho}$ , where  $r$  is the radius of the tube and  $t$  its thickness. Using the following parameter values as described in [13,28],  $r \sim 10^{-3} \text{ m}$ ,  $t \sim 10^{-4} \text{ m}$ ,  $E \sim 1 \text{ kPa}$ ,  $\rho \sim 10^3 \text{ kg m}^{-3}$ , we find that  $c_{\text{model}}/c_{\text{bird}} \sim 30$ , and explaining the discrepancies in wave velocities observed. However, we note that within our experimentally observed paradigm of sound production via solitary pulses, the fundamental frequency is set by the time taken for the solitary pulse to reach the mouth from its point of origin, which would be the posterior end of the MVM for the bird and the location of the probe for the device, i.e. it scales as  $c/l$ , where  $l$  is the length of the vibrating structure. Hence although the device is much longer, the wave speed is also larger, which results in a similar fundamental frequencies for both real and mimicked structures. This implies that in spite of the morphological simplifications and relatively larger size of our device, we may still capture the essential biophysics of song production, and match what is observed in nature up to scaling factors.

### 2.3. Controllable biomimetic birdsong

Using a temporally programmed indentation profile  $h(t)$  into the probe allows us to sweep through frequency (electronic supplementary materials, figure S5), and figure 2*a* shows how sound with high harmonic content can be obtained with large successive frequency sweeps obtained by this method. We first position the motor so that the tube is poised at the onset of phonation and then use the motor as a switch that can be turned on or off rapidly. Thus, transitions between sounds can be triggered dynamically by choosing the bias points appropriately. In figure 2*b,c*, we show that it is possible to switch the acoustic behaviour of the device from generating solitary, pulsed to chaotic modes of vocalization as the motor is dynamically activated to move through the different phases that we delineated previously (electronic supplementary materials, Audio S1).

To demonstrate the versatility of the biomimetic syrinx, we chose to mimic three characteristic songs from three species (electronic supplementary material, Audio): vireo songs that are mostly tonal, bengalese finches that have songs with notes of both high and single harmonic content, and zebra finch songs that have notes with very high harmonic content and complicated transitions. To create the song of the vireo, the motorized probe was placed close to the mouth of the device to ensure operation in the tonal regime, and its



**Figure 2.** Controllable transitions in a biomimetic syrinx. (a–c) Dynamic action of probe. (a)(i) and (b)(i) The spectrograms of two complicated sounds created by dynamically controlling the probe. The probe is programmed with a pulse sequence to create the sounds (we specify the position and acceleration). (a)(ii) and (b)(ii) The position of the probe, relative to the spectrogram of the sound created. (c)(i)–(iii) The sound pressure levels for the sections indicated in (b)(ii). (c)(i) Sinusoidal, (ii) oscillations following a period-doubling transition, and (iii) chaotic sound. We see that by actuating the probe, we can change the qualitative nature of sound production. (d–f) Spectrograms of real and mimicked bird songs (using dynamic action of the probe). (d)(i) A song from a red-eyed vireo, and (d)(ii) songs mimicked by dynamical control of the device. The vireo has beautiful tonal songs with rapid and large variations in frequency. The device was operated in the tonal regime and reproduces songs well when the variation in frequency is not very large. (e)(i) A real and (e)(ii) mimicked song from a bengalese finch. These songs consisted of short harmonic pulses, which were well reproduced by the device. Even subtle frequency changes and transitions to a period doubled mode were well captured. (f)(i) A real and (f)(ii) mimicked song from a zebra finch. These songs show large spectral variation as well as high harmonic content and are difficult to reproduce. However, given the limitations of the linear motor, many features of the zebra finch song (large changes in frequency, transitions to chaos, high harmonic content) can be qualitatively reproduced. (Online version in colour.)

depth was programmed to change  $\omega(t)$ , the local fundamental frequency. The two spectrograms (electronic supplementary materials, Audio S2) show that this simple principle can be used to achieve a high degree of similarity of the mimicked song to the original; figure 2d confirms this visually by comparing the spectrogram of a recreated song of a vireo along with the original. To create the song of the bengalese finch, the motor was placed further behind the mouth to enable both tonal and pulsed sounds. The first section of the song shows pulses with large harmonic content created using the probe as a switch, rapidly moving to a depth commensurate with the fundamental frequency of the song and then rapidly retracting it. The first two syllables show a period doubled sound created by the device (electronic supplementary material, Audio S3). The second part of the song shows tonal structure much like that of a vireo song, and can be

recreated in a similar fashion; a spectrogram of the characteristic pulsed song of a bengalese finch and its reproduction using our device is shown in figure 2e. Finally, we turn to the zebra finch song that combines the high-speed pulses of a bengalese finch with sweeps in tonal songs and rapid transitions from tonal to multiharmonic or from multiharmonic to chaotic. To create these notes the motor was placed even further behind the mouth; tonal to multiharmonic sweeps were generated using a tonal to solitary transition and from solitary to chaotic transition to generate multiharmonic to chaotic sweeps. The recreated song bears a qualitative resemblance to the original but is not exact because of limitations on the maximum acceleration of the linear probe. The song shows three distinct features; sweeping transitions from single to multiharmonic sounds (1 and 3 in figure 2f), a sweeping transition to chaotic sounds (2 in figure 2f) and

multiharmonic sweeps to harmonic stacks (4 and 5 in figure 2f). The recreated spectrogram shown in figure 2f (electronic supplementary material, Audio S4) shows oscillations in the frequency especially for the last notes due to ringing of the linear motor itself and could not be avoided as the motor was moving close to its maximum acceleration.

### 3. Mathematical model for tracking a tonal song

Having shown that our biomimetic device can effectively mimic a range of songs, a natural question that arises is that of quantifying the acoustics of sound and song production, and its control. The complex elasto-hydrodynamics of phonation and the ensuing nonlinear dynamics of fluid-structure interaction is a difficult problem in general. But is it possible to capture the dynamics of a simple song and consider its control? The modular nature of operating the syringe is such that we can bring the system close to threshold slowly and then excite it rapidly. In the case of a tonal song, such as that of the vireo, this is tantamount to exciting a single frequency that is changed, so that the system moves from a silent regime to a spontaneously oscillating one, and back. Here we couch this as a problem of determining how to move in between these regimes to track a given song, to mimic the vocal response of a finch son to that of his father's birdsong. When our system is close to the threshold for self-excited oscillations, it may be minimally modelled as an oscillator with a dynamically varying spring stiffness that allows for a controllable frequency  $\omega(t)$  and a controllable damping coefficient  $r(t)$  whose sign changes as the system transitions through the threshold for instability, a linearized variant of the nonlinear oscillator models used to mimic birdsong [20], but now couched in an optimization framework. The equation of motion for a harmonic oscillator with time varying stiffness and damping can be written as

$$\ddot{x} + r(t)\dot{x} + \omega^2(t)x = 0, \quad (3.1)$$

where  $x(t)$  models the acoustic signal corresponding to the song, and  $r(t)$  and  $\omega(t)$  are two functions that the bird manipulates in order to generate the signal. In reality, the frequency is effectively controlled by the static and dynamic tube tensions, while the damping is controlled by the flow rate. Given a target bird song as a time series  $u(t)$ , we ask to determine slowly varying functions  $r(t)$  and  $\omega(t)$ , for which the solution of (3.1) comes close to the target, formulated as the solution of the following problem: find  $r(t)$ ,  $\omega(t)$  subject to (3.1) that minimizes the global error

$$\int_0^T (x - u(t))^2 + W_1 \left( \frac{d\omega}{dt} \right)^2 + W_2 \left( \frac{dr}{dt} \right)^2 dt. \quad (3.2)$$

Here the first term penalizes deviations in the produced song, while large weights  $W_1$  and  $W_2$  suppress rapid variations in  $\omega(t)$  and  $r(t)$  (even though  $\omega$  itself can be large, and  $r$  itself can be very small), and the harmonic oscillator equation serves as a minimal model near the onset of phonation. When these parameters vary slowly, we take advantage of the WKB approximation [36] for the solution of (3.1), which allows the approximate solution to (3.1) to be written as

$$x(t) = \frac{1}{\sqrt{\omega(t)}} (A e^{-\Psi(t) + i\Phi(t)} + B e^{-\Psi(t) - i\Phi(t)}), \quad (3.3)$$

where  $\Psi(t) = \int_0^t (r(s)/2) ds$  and  $\Phi(t) = \int_0^t \omega(s) ds$ , and  $A$  and  $B$  are arbitrary constants determined by initial conditions. Substituting these results in (3.2), we convert the differential optimization algorithm to an algebraic one by dividing the duration of the bird song into  $N$  intervals, with  $r$  and  $\omega$  approximated to be constants in each interval but varying from interval to interval, and  $A$  and  $B$  determined so that  $x$  and  $\dot{x}$  are continuous across intervals. The global optimization problem is then converted into a sequence of  $N$  local optimization problems, where for the  $k$ th local optimization problem, the values of  $r_k$  and  $\omega_k$  are determined by the solution of the following problem:

$$\text{Min}_{r_k, \omega_k} \int_{t_{k-1}}^{t_k} (x - u(t))^2 + \tilde{W}_1 (\omega_k - \omega_{k-1})^2 + \tilde{W}_2 (r_k - r_{k-1})^2 dt,$$

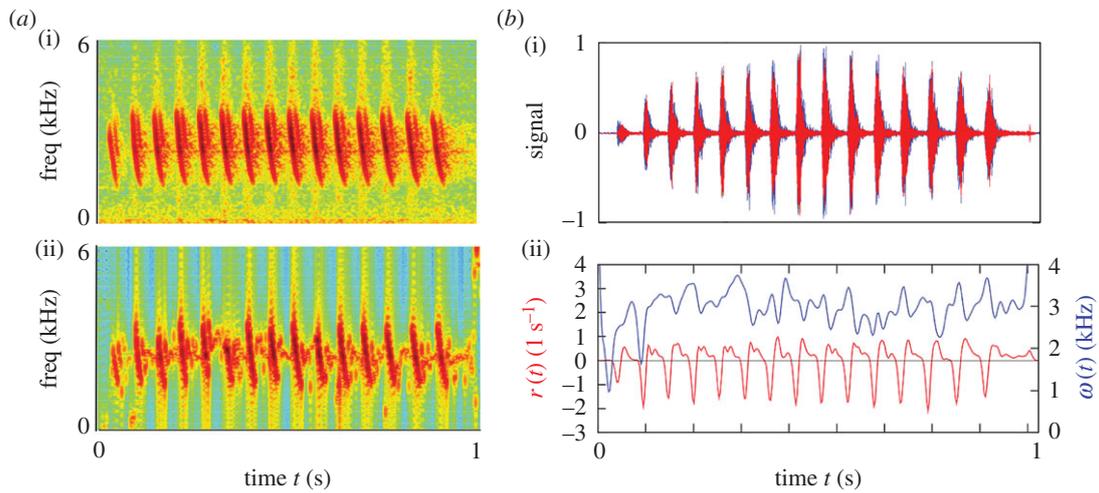
where  $x(t)$  is given by (3.3) by assuming  $r(t) = r_k$  and  $\omega(t) = \omega_k$ , and  $\tilde{W}_{1,2}$  are the rescaled weights  $W_{1,2}$ , respectively. Our simulations that were carried out using a steepest descent algorithm (Nelder-Mead) implemented in Matlab using a step size of 0.1; our results were insensitive to the choices of the weights in the regime  $O(100-1000)$ .

In figure 3a(i),(ii), we show the spectrograms of a real and simulated vireo song that shows good agreement, as confirmed in figure 3b(i), which shows the actual time trace  $x(t)$  that sits on top of the target  $u(t)$ . In figure 3b(ii), we also show the time-dependent frequency  $\omega(t)$  and the damping  $r(t)$  derived from the solution of the optimization problem, noting that they vary very slowly, and are thus viable controls (electronic supplementary material, Audio S5,S6). We thus see that it is possible to capture the entire dynamics of the song using a linear oscillator model with slowly varying frequency and damping, consistent with our experimental study.

### 4. Discussion

Birdsong has all the hallmarks of complex behaviour in animals, as it exhibits the ability of the bird to exquisitely control a complex physical system, the syringe, to realize the task of learning mimicry with gradual variations. The creation of song involves two separate processes. The first is the process of sound production itself via the onset of vibratory modes due to coupling between the membrane and aerodynamically generated forces. The second is the process of song control via motion through the phase space of parameters associated with frequency and amplitude modulation. Sound production via the MEAD mechanism, wherein dynamic and asymmetric membrane opening and closing leads to self sustained oscillations, has been verified in both oscine and non-oscine species [16]. However, the case for song control is still up for debate. Current *in vivo* studies measure correlations between fundamental frequency and muscle stimulation [37] but do not provide insight into details of the membrane tension or air sac pressure as a function of time. Our simple estimates suggest that controlling song production by varying these parameters requires large and rapid changes in membrane tension which have not been demonstrated experimentally. Furthermore, the effect of muscular forces on the membrane remains unclear, and, in particular, how it modulates tension on slow and fast time scales remains unknown.

Here, we have attempted to study the biomechanical aspects of the process by focusing on creating a controllable biomimetic syringe to produce song. Following earlier work,



**Figure 3.** Mimicking a tonal song of the red-eyed vireo using a minimal model described in §3. (a) Spectrogram of a song of a red-eyed vireo: (i) original and (ii) computational mimic (see text). (b)(i) Time trace of the amplitude of the center frequency of the tonal song for the actual birdsong (blue) and its computational mimic (red) lie essentially on top of each other. (ii) The solution of the optimization problem (3.2) shows the temporal evolution of the damping  $r(t)$  (left abscissa) and the frequency  $\omega(t)$  (right abscissa). (Online version in colour.)

we have implemented this using an elasto-hydrodynamic instability associated with flow through a simple rubber tube, such that we can bring the system close to an excitability threshold using a combination of a static stretch and flow. A dynamic probe then allowed us to dynamically trigger phonation patterns to create the basic syllables of simple birdsongs. The advantage of this approach is that it allows for a slow, coarse actuator that brings the system close to an instability and then a using a fast, fine actuator to transition easily across the excitability threshold. The resulting behaviour of our spatially extended acoustic oscillator allows for a range of physiological outputs with a minimal controller. By placing the linear motor at different lengths along the tube, we further showed that songs from different species can be reproduced, suggesting the ease with which a simple avian vocal tract can mimic a range of songs and sounds. This strategy of localized control is qualitatively consistent with early observations made in [12], and later summarized in [13], that show that small medial rotations of the third bronchial ring that insert the lateral labium into the bronchial lumen can produce large changes in the fundamental frequency, without stretching the ML or MTM in a manner similar to the mechanical probe used in our study. Finally, we have shown how a minimal mathematical model that takes the form of an excitable harmonic oscillator with slowly varying damping and

stiffness can be used to study how an optimization approach can mimic birdsong.

Overall, our study seeks to emphasize that the complexity of the overall task of producing birdsong can be broken down into a set of simpler biomechanical and control tasks that may not be so difficult to accomplish individually. This suggests that by harnessing the physical substrate of a soft tube susceptible to self-excited oscillations, developing organisms and perhaps evolution itself might have stumbled on using simple instabilities to produce and control birdsong. More broadly, our study adds to the growing realization that physical instabilities with rich nonlinear dynamics, when coupled to relatively simple control mechanisms, may provide a mechanism for organisms to begin to create complex behaviour by taking advantage of their physical, material nature.

**Data accessibility.** The datasets supporting this article have been uploaded as part of the supplementary material.

**Competing interests.** We declare we have no competing interests.

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