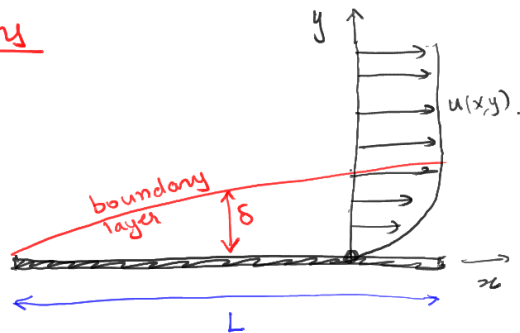


# Prandtl boundary layer equations

Characteristic scales:  $x, y, u, v, p, t$  steady

$x \sim L, y \sim \delta, u \sim U, v \sim V$

$p \sim P$  or  $\rho U^2$ .



$$\rho \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

neglect  $\delta/L \ll 1$

$$\frac{\delta}{L} \ll 1$$

$$P = \rho U^2$$

$$\frac{\mu U}{\delta^2} = \frac{\rho U^2}{L} \Rightarrow \delta = \sqrt{\frac{\mu L}{\rho U}}$$

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\rho U^2}{L} = \frac{\rho U V}{\delta} \quad \left( \frac{P}{L} \right)$$

$$\frac{\mu U}{L^2} \quad \left( \frac{\mu U}{\delta^2} \right)$$

neglect

$$\frac{\partial p}{\partial y} = 0$$

$$\rho \left[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

neglect  $\delta/L \ll 1$

$$\frac{\rho U V}{L} \quad \frac{\rho V^2}{\delta} \quad \frac{P}{\delta} \quad \left( \frac{P}{\delta} \right)$$

$$\frac{\mu V}{L^2} \quad \frac{\mu V}{\delta^2}$$

neglect  $\delta \ll L$

$$\left( \frac{\delta}{L} \right)^2 = \frac{\mu}{\rho U L}$$

$$\frac{\rho U^2}{\delta} \left( \frac{\delta}{L} \right)^2 \ll \frac{P}{\delta}$$

$$P \sim \rho U^2, \quad \left( \frac{\delta}{L} \right)^2 = \frac{\mu}{\rho U L} \quad \star$$

$$\frac{P}{\delta} \sim \frac{\rho U^2}{\delta} \Rightarrow \frac{\mu \cdot U}{\delta \cdot L}$$

$$\frac{U}{L} = \frac{V}{\delta} \Rightarrow V = U \left( \frac{\delta}{L} \right)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{U}{L} = \frac{V}{\delta}$$

Prandtl Boundary layer eqns.

$$\left\{ \begin{aligned} \rho \left[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] &= -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} \\ 0 &= -\frac{\partial p}{\partial y} \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \end{aligned} \right.$$

... x-mom

... y-mom

... incompressibility.

$$\delta = L \cdot \left( \frac{\mu}{\rho U L} \right)^{1/2}$$

$$\delta \sim \sqrt{\frac{\nu x}{U}} \text{ from handwaving argument}$$