

INITIAL AND BOUNDARY CONDITIONS.

RECAP: The Navier-Stokes equations

$$\text{Momentum: } \rho \left[\frac{\partial \underline{u}}{\partial t} + (\underline{u} \cdot \nabla) \underline{u} \right] = -\nabla p + \rho \underline{g} + \mu \nabla^2 \underline{u}$$

$$\underline{u} = (u, v, w)$$

$$\text{Mass: } \nabla \cdot \underline{u} = 0$$

$$\underline{x} = (x, y, z)$$

Rules for evolving $\underline{u}(\underline{x}, t)$.

Initial: $\underline{u}(\underline{x}, t=0) = \underline{u}_0$ for $\underline{x} \in \Omega$. where $\Omega =$ fluid domain.

Boundary conditions:

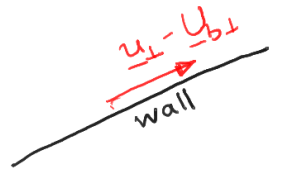
1. No SLIP: $\underline{u}(\underline{x}, t) = \underline{U}_b(\underline{x}, t)$ for $\underline{x} \in \partial\Omega$.

Applies to viscous fluids next to solid walls.

2. Free slip a.k.a. no penetration:

$$\underline{u}_\perp(\underline{x}, t) = \underline{U}_{b\perp}(\underline{x}, t) \quad \text{for } \underline{x} \in \partial\Omega$$

Applies to inviscid fluids.



3. Kinematic boundary condition (for moving material boundaries)

$$\frac{D\underline{x}}{Dt} = \underline{u}(\underline{x}, t) \quad \text{for } \underline{x} \in \partial\Omega.$$



4. Dynamic boundary condition.

$$\underline{T} \cdot \hat{n} = \underline{t} \quad \text{specified on } \underline{x} \in \partial\Omega$$

↙ unit normal to $\partial\Omega$.

$\underline{T} \cdot \hat{n} =$ force per unit area on a surface w/ normal \hat{n} .