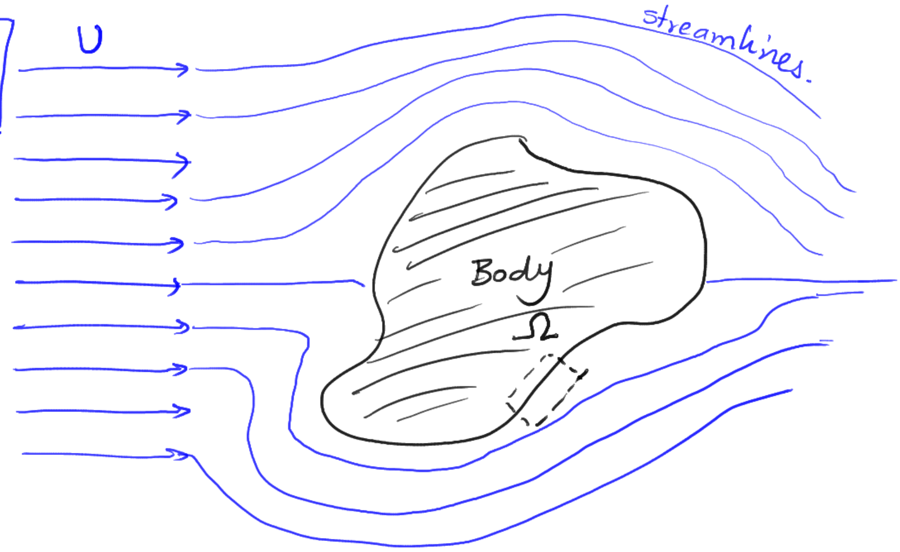
FORCE ON IMMERSED BODY   § 6.5

Recap: (i) Potential flow: $\underline{u} = \nabla\phi$

(ii) Bernoulli: $p + \rho\left[\frac{\partial\phi}{\partial t} + \frac{1}{2}|\underline{u}|^2\right] - \rho\underline{g}\cdot\underline{x} = f(t)$ alone.

$$\boxed{w = Uz + \frac{Q - i\Gamma}{2\pi}\log z + \frac{a_1}{z} + \frac{a_2}{z^2} + \cdots}$$



- Inviscid fluid $\Rightarrow \underline{\underline{T}} = -p\underline{\underline{I}}$.

$$\underline{F} = \int_{\partial\Omega} -p\hat{n}\, dA$$

1. Contribution from hydrostatic pressure.

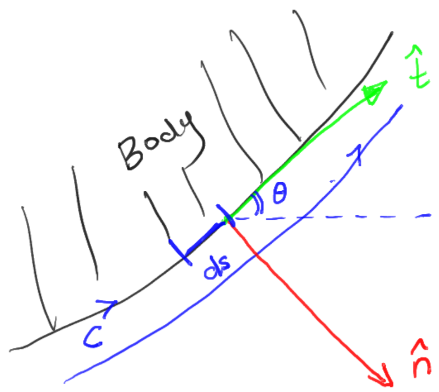
$$\underline{F}_h = \int_{\partial\Omega} -p\hat{n} = \int_{\partial\Omega} +(\rho\underline{g}\cdot\underline{x})\,\hat{n}\, dA = \rho\underline{g}|\Omega| \quad \ldots$$ Archimedes principle.

2. Steady potential flow: $p = -\frac{1}{2}\rho|\underline{u}|^2$.

$$\underline{F}_s = \int_{\partial\Omega} +\frac{1}{2}\rho|\underline{u}|^2\,\hat{n}\,\underline{\underline{dA}}$$

$$u - iv = w'(z) = |\underline{u}|\cdot e^{-i\theta}$$

$$|\underline{u}| = w'(z)\,e^{i\theta} \Rightarrow |\underline{u}|^2 = w'(z)^2 e^{2i\theta}.$$



$$F_x - iF_y = \int_{\partial\Omega} +\frac{1}{2}\rho|\underline{u}|^2\left(ie^{-i\theta}\right) dz\, e^{-i\theta}$$

$$= \oint_C \frac{i}{2}\rho\, w'(z)^2 \left[e^{2i\theta}(e^{-i\theta})(e^{-i\theta})\right]dz$$

$$= \oint_C \frac{i}{2}\rho\, w'(z)^2\, dz.$$

$\hat{t} = e^{i\theta}$

$\hat{n} = -ie^{i\theta}$.

$dz = ds\, e^{i\theta}$.

$ds = dz\, e^{-i\theta}$.

$$w'(z) = U + \frac{\overset{0,\text{ mass conservation.}}{\cancel{Q}} - i\Gamma}{2\pi z} - \frac{a_1}{z^2} - \frac{2a_2}{z^3}\cdots$$

$$w'(z)^2 = -\frac{iU\Gamma}{\pi z} + U^2 - \frac{\Gamma^2}{4\pi^2 z^2} + \text{higher power of } \frac{1}{z}.$$

$$F_x - iF_y = \frac{i\rho}{2}\cdot(2\pi i)\left(-\frac{iU\Gamma}{\pi}\right)$$

$$\boxed{F_x - iF_y = i\rho U\Gamma.}$$

$\boxed{F_x = 0,}$

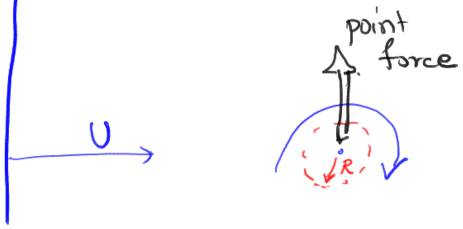
No drag!

D'Alembert paradox.

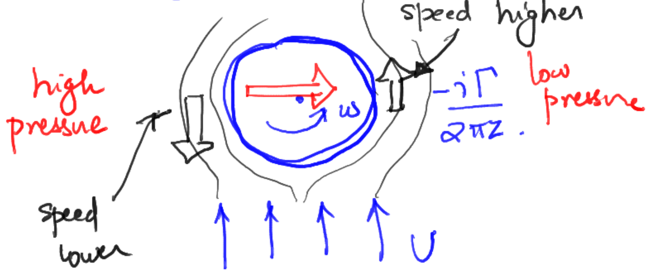
$\boxed{F_y = -\rho U \Gamma}$

Kutta-Joukowsky theorem.

Examples sheet 3(?)



<u>Magnus effect</u>



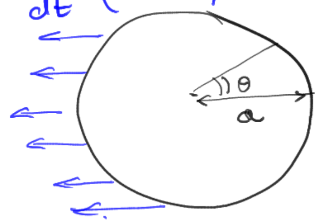
3. Added mass because of unsteady term.

$$\underline{F}_u = \int_{\partial\Omega} +\rho \frac{\partial\phi}{\partial t} \hat{n} \, dA$$

$$W(z) = U\left(z + \frac{a^2}{z}\right) \quad \ldots \text{ flow around a circle.}$$

$$\phi = \mathrm{Re}(w) = U\left(r + \frac{a^2}{r}\right)\cos\theta. \Rightarrow \frac{\partial\phi}{\partial t} = \frac{dU}{dt}(a+a)\cos\theta = 2a\cos\theta \frac{dU}{dt}$$

$$\underline{F}_u = \int_{\partial\Omega} \rho \left.\frac{\partial\phi}{\partial t}\right|_{r=a} \cdot (\cos\theta\, \hat{e}_x + \sin\theta\, \hat{e}_y)\, a\, d\theta$$



$$= \int_0^{2\pi} \rho \frac{dU}{dt}(2a\cos\theta) \cdot (\cos\theta\, \hat{e}_x + \sin\theta\, \hat{e}_y)\, a\, d\theta.$$

$$= \underbrace{2\pi a^2 \cdot \rho}_{\substack{\text{mass of a certain} \\ \text{amount of fluid} \\ \text{" Added mass}}} \underbrace{\frac{dU}{dt}}_{\text{acceleration}} \cdot \hat{e}_x.$$

Summary

(i) Buoyancy

→ (ii) Steady term { Kutta-Joukowsky / D'Alembert Paradox

(iii) Added mass.