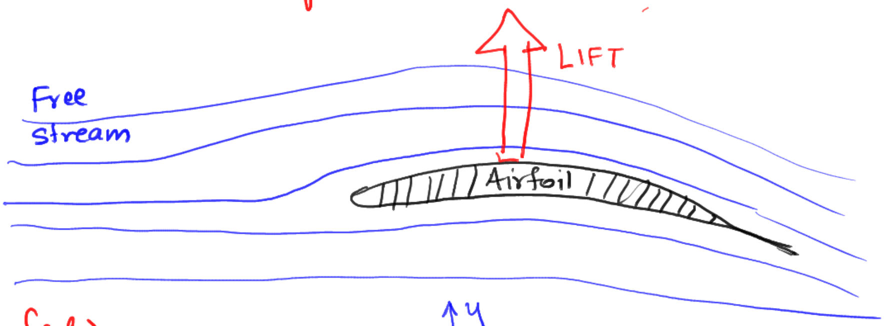
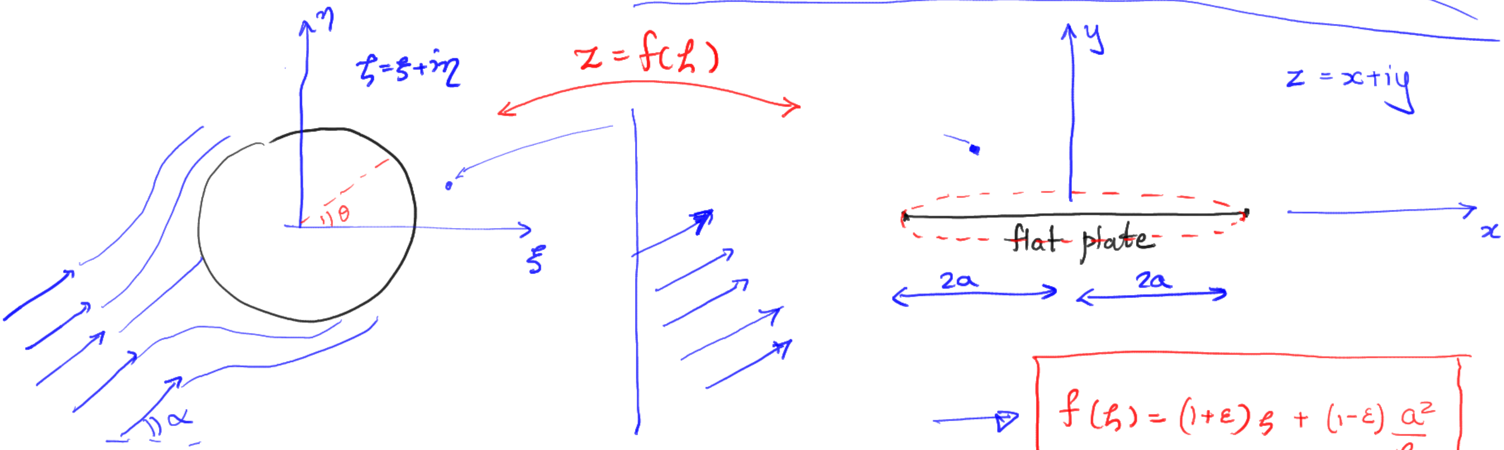
Flow around an airfoil §6.6.

Recap: All of potential.



Conformal mapping



$$\boxed{f(\zeta) = (1+\varepsilon)\,\zeta + (1-\varepsilon)\,\frac{a^2}{\zeta}.}$$

$$w(\zeta) = U\left(\zeta e^{-i\alpha} + \frac{a^2}{\zeta e^{-i\alpha}}\right) - \frac{i\Gamma}{2\pi}\log z.$$

$$W(z) = w\left(f^{-1}(z)\right) \quad \text{analytic complex potential}$$

Say $\zeta = a e^{i\theta} \Rightarrow f(\zeta) = (1+\varepsilon)\,a e^{i\theta} + (1-\varepsilon)\,a e^{-i\theta}$

$= 2a\cos\theta + 2ia\varepsilon\sin\theta.$

$z = f(\zeta)$ is an ellipse

- major axis $2a$ along x
- minor axis $2a\varepsilon$ along y

Limit $\varepsilon \to 0$. $\qquad \varepsilon > 0 \Leftrightarrow$ bijection for $|\zeta| \geq a$ to z.

$$u - iv = \frac{dW}{dz} = \frac{dw/d\zeta}{dz/d\zeta} = \frac{U\left(e^{-i\alpha} - \frac{a^2}{\zeta^2}e^{i\alpha}\right) - \frac{i\Gamma}{2\pi\zeta}}{\left(1 - a^2/\zeta^2\right)}.$$

Trailing edge $\zeta = a$. (insist no singularity there). "Kutta condition".

Numerator. $\underbrace{U\left(e^{-i\alpha} - \frac{\cancel{a^2}}{\cancel{a^2}}e^{i\alpha}\right)}_{-2Ui\sin\alpha} - \frac{i\Gamma}{2\pi a} = 0. \Rightarrow \boxed{\Gamma = -4\pi a U\sin\alpha}$

Force on the flat plate: $F_y = -\rho U\Gamma = \rho U\left(4\pi a U\sin\alpha\right)$

$= \rho U^2(4a)\ \pi\sin\alpha.$ ... lift on a flat plate (airfoil).

Coefficient of lift $C_L = \dfrac{F_y}{\frac{1}{2}\rho U^2 \underbrace{(4a)}_{\text{chord}}} = 2\pi\sin\alpha.$

$$\boxed{C_L(\alpha) = 2\pi\sin\alpha}$$