# Derived potential flows

§ 6.3.4 & 6.3.5.

Recap: Uniform flow – $W(z) = Ue^{-i\alpha} z$.

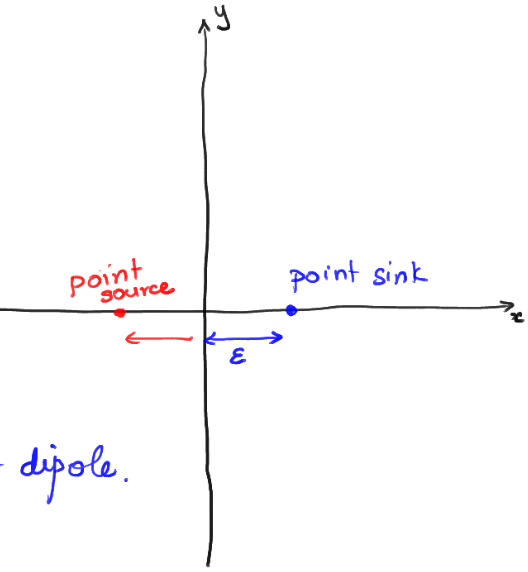
Point source – $W(z) = \dfrac{Q}{2\pi} \log z$

Point vortex – $W(z) = \dfrac{i\Gamma}{2\pi} \log z$.

Point dipole –

Higher multipoles –

1. Point dipole.




$$W(z) = \lim_{\varepsilon \to 0} \left[ \frac{-Q}{2\pi} \log(z-\varepsilon) + \frac{Q}{2\pi} \log(z+\varepsilon) \right]$$

$Q = \dfrac{D}{2\varepsilon}$ . ⟵ $D$ = strength of the point dipole.

$$W(z) = \lim_{\varepsilon \to 0} \frac{D}{2\pi} \left[ \frac{-\log(z-\varepsilon) + \log(z+\varepsilon)}{2\varepsilon} \right] = +\frac{D}{2\pi z}$$

$$W(z) = \frac{D}{2\pi z} = \frac{D e^{-i\theta}}{2\pi r} \Rightarrow \phi = \frac{D\cos\theta}{2\pi r} \qquad z = x + iy$$

$$\psi = -\frac{D\sin\theta}{2\pi r}. \qquad z = re^{i\theta}$$

Velocity $\underline{u} = \nabla\phi$

$$u_r = \frac{\partial\phi}{\partial r} = -\frac{D\cos\theta}{2\pi r^2}$$

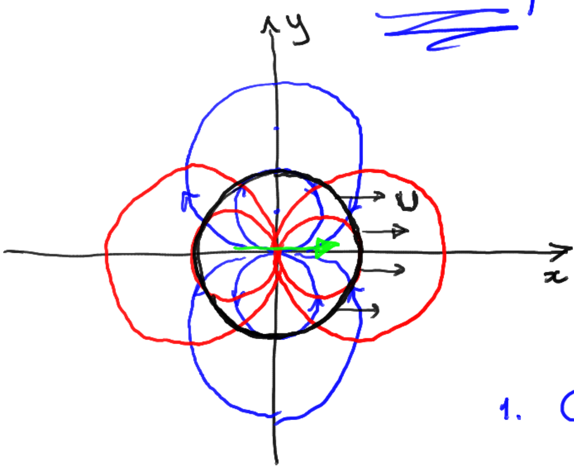
$$u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{D\sin\theta}{2\pi r^2}$$

Streamlines: $\psi = \text{const}$

$$\frac{D\sin\theta}{2\pi r} = \frac{D}{4\pi c} \Rightarrow r = 2c\sin\theta.$$

$$\sqrt{x^2+y^2} = \frac{2cy}{\sqrt{x^2+y^2}} \Rightarrow x^2+y^2-2cy = 0$$

$$\boxed{x^2 + (y-c)^2 = c^2}$$




Equipotential lines: $\phi = \text{const}$

$$\boxed{(x+c)^2 + y^2 = c^2.}$$

1. Can $D$ be complex?

2. What about point dipole of point vortices?

3. What is this flow?

@ $r = a \Rightarrow u_r = \left(\dfrac{-D}{2\pi a^2}\right)\cos\theta$

$u_\theta = \left(\dfrac{-D}{2\pi a^2}\right)\sin\theta$

Uniform motion of circle w/ speed $U$ along $x$-axis.

$\underline{V} = U\hat{e}_x$.

$V_r = U\hat{e}_x \cdot \hat{e}_r = U\cos\theta$

$V_\theta = U\hat{e}_x \cdot \hat{e}_\theta = U\sin\theta$.

If $\boxed{D = -2\pi a^2 U}$ $\Rightarrow$ $u_r = V_r$. $\Leftarrow$ no penetration condition satisfied.

$u_\theta \neq V_\theta$ $\Leftarrow$ no slip condition NOT satisfied.

- <u>Higher multipoles:</u>

  - $W_{dipole}(z) = \frac{d}{dz} W_{source}(z)$.
  - Point quadrupoles = point dipole of point dipoles. $W(z) = q/2\pi z^2$
  - Point octupole = point dipole of point quadrupoles. $W(z) \propto 1/z^3$

  ⋮

  and so on.

$$W(z) = \underbrace{a_0 \log z}_{\text{point source + vortex}} + \underbrace{\frac{a_1}{z}}_{\text{point dipole}} + \underbrace{\frac{a_2}{z^2}}_{\text{point quadrupole}} + \cdots \quad \text{multipole expansion.}$$

e.g. $a_1 = -a^2 U$ corresponds to flow generated by a uniformly translating circle.