CONTINUUM APPROXIMATION

The assumption that matter is infinitely divisible is called the continuum approximation.

Quantitative definition

- Matter is made of atoms and molecules.

  ℓ = mean free path of molecules

- The flow varies over a length scale

  L = length scale of the flow

> Mean free path is the length a molecule travels between collisions with other molecules.

e.g $\ell \approx 1\ \mu m$ to $1\ nm$

$L \approx 1\ cm$ to $10\ m$

$\frac{\ell}{L} \approx 10^{-4}$ to 10^{-10}

> Continuum approximation valid if $\frac{\ell}{L} \ll 1$.

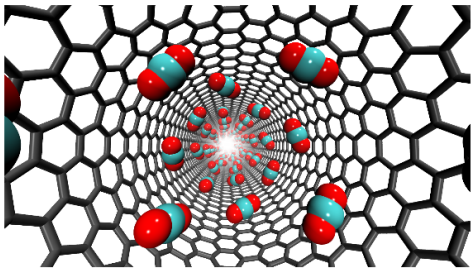
Examples

1. Diameter of carbon nanotube = $0.4\ nm = L$

   Mean free path $\ell \approx 0.25m$

   $\frac{\ell}{L} = \frac{0.25}{0.4} \approx 0.625$. (not small)

   Continuum hypothesis likely not applicable.



Water through carbon nanotubes

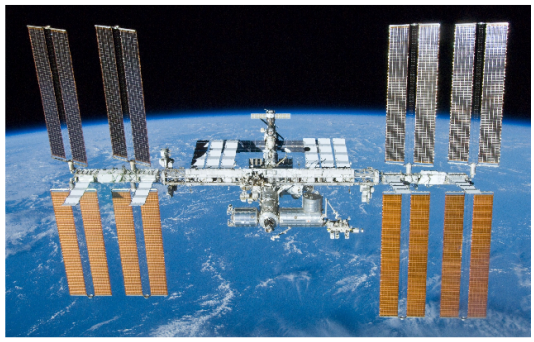
2. Orbits 400 km above mean sea level.

   ℓ = mean free path = 20 km

   $L = (79\ m \times 109\ m) \approx 100\ m$

   $\frac{\ell}{L} = \frac{20\ km}{100\ m} \simeq 200$ definitely not small.

   Continuum hypothesis not applicable.



International space station

A dimensionless number:

Knudsen number = $\frac{\ell}{L}$ ... devoid of dimensions

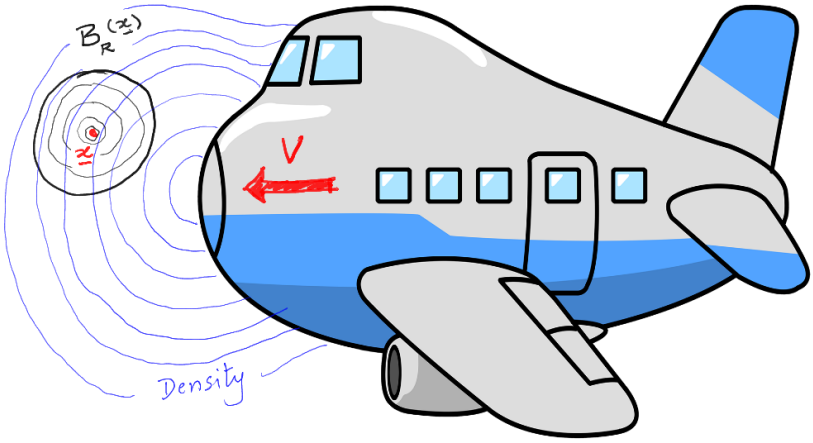
Consequence of continuum hypothesis : Continuum fields

Average Density

$$\langle \rho \rangle_{\underline{x},R} = \frac{M_{\underline{x},R}}{V_{\underline{x},R}}$$

$M_{\underline{x},R}$ = mass of fluid in $B_R(\underline{x})$

$V_{\underline{x},R}$ = volume of $B_R(\underline{x}) = \frac{4}{3}\pi R^3$.



Source: pixy.org

$$\rho(\underline{x}) = \lim_{R \to 0} \langle \rho \rangle_{\underline{x},R} .$$

$\frac{l}{L} \ll 1$