CONSERVATION OF ANGULAR MOMENTUM

$$\frac{\partial b}{\partial t} + \frac{\partial}{\partial x_j}(b u_j) = q^B + \frac{\partial T^B_{\cdot\cdot j}}{\partial x_j} \qquad \ldots \text{ differential form only.}$$

Density of angular momentum : $\underline{b} = \rho\, \underline{x} \times \underline{u}$ or $b_i = \rho\, \epsilon_{ijk} x_j u_k$.

Volumetric source : $\underline{q}^B = \underline{x} \times \rho \underline{g}$ or $q^B_i = \rho\, \epsilon_{ijk} x_j g_k$.

Surface source : $\underline{T}^B = \underline{x} \times \underline{T}$ or $T^B_{im} = \epsilon_{ijk}\, x_j\, T_{km}$

(Here $\underline{T}$ is the Couchy stress tensor from conservation of momentum.)

$$\frac{\partial}{\partial t}\left(\rho \epsilon_{ijk} x_j u_k\right) + \frac{\partial}{\partial x_m}\left(\rho \epsilon_{ijk} x_j u_k u_m\right) = \epsilon_{ijk}\, \rho\, x_j g_k + \frac{\partial (\epsilon_{ijk} x_j T_{km})}{\partial x_m}$$

momentum conservation

$$\epsilon_{ijk} x_j \cancelto{0}{\left[\frac{\partial}{\partial t}(\rho u_k) + \frac{\partial}{\partial x_m}(\rho u_k u_m) - \rho g_k - \frac{\partial T_{km}}{\partial x_m}\right]} + \left[\epsilon_{ijk}\, \rho u_k u_m - \epsilon_{ijk} T_{km}\right] \overbrace{\frac{\partial x_j}{\partial x_m}}^{\delta_{jm}} = 0$$

$$\cancelto{0}{\epsilon_{ijk}\, \rho u_k u_j} - \epsilon_{ijk} T_{kj} = 0 \quad \Longrightarrow \quad \boxed{\epsilon_{ijk} T_{kj} = 0}$$

(symmetry)

OR

$$\boxed{T_{jk} = T_{kj} \quad \text{for all } k, j}$$

In words, the stress tensor is symmetric.